Topic 3: Oscillators, waves, flows

Content:

- harmonic oscillator
- damped oscillator
- driven oscillator
- waves
- Huygens principle, Doppler effect
- flow of liquids and air

basic terms and quantities

The general study of the relationships between motion, forces, and energy is called **mechanics**.

Motion is the action of changing location or position.

Motion may be divided into three basic types - translational, rotational, and oscillatory.

The study of motion without regard to the forces or energies that may be involved is called **kinematics**. It is the simplest branch of mechanics.

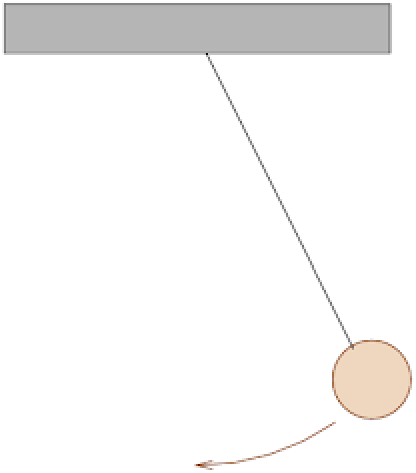
The branch of mechanics that deals with both motion and forces together is called **dynamics** and the study of forces in the absence of changes in motion or energy is called **statics**.

basic terms and quantities

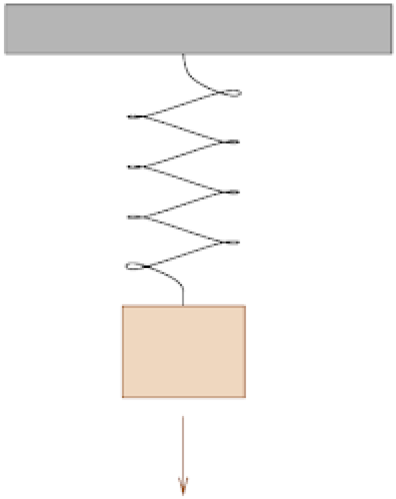
Motion is the action of changing location or position.

Motion may be divided into three basic types - translational, rotational, and oscillatory.

Oscillation is the repetitive variation, typically in time, of some measure about a central value (often a point of equilibrium) or between two or more different states.



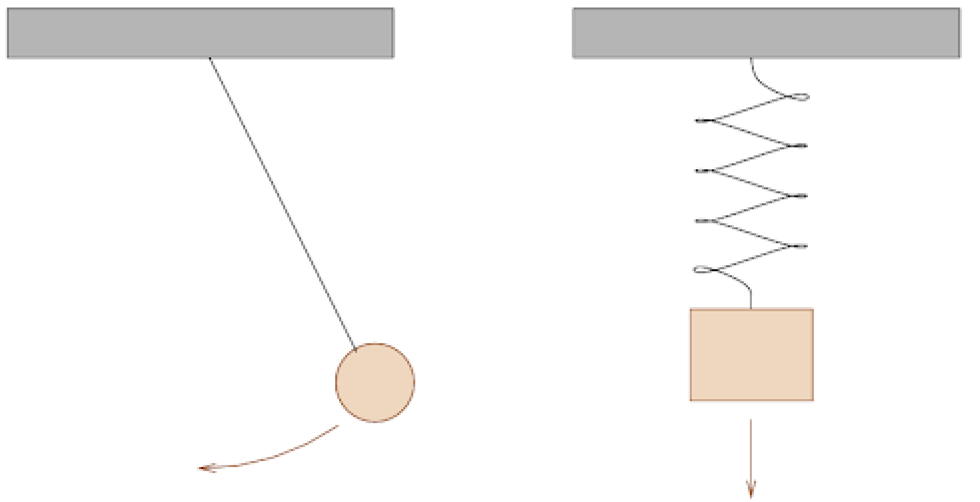
mathematical pendulum



oscillating spring with a mass

basic terms and quantities

Oscillation is the repetitive variation, typically in time, of some measure about a central value (often a point of equilibrium) or between two or more different states.



Basic quantitites:

T – period [s] (T – time for one complete cycle)

$f = 1/T$ – frequency [s^{-1} = Hz] (f – number of oscillations in 1 second)

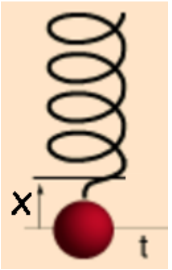
$\Omega = 2\pi f$ – angular frequency [$rad \cdot s^{-1}$]

HARMONIC OSCILLATOR

- **Simple harmonic oscillator -** F is the only acting force
- **Damped oscillator –** Friction (damping) occurs
- **Driven oscillator –** Damped oscillator further affected by an external force

1. Simple harmonic oscillator

an oscillating spring with a mass



x – distance,
t – time,

Balance of the system is given by (with help of Newton’s second law and Hooke’s law):

a) $$F = ma = m\frac{d^2x(t)}{dt^2}$$

Newton’s second law („Law of Power“)

b) $$F = -kx(t)$$

so called Hooke’s law:
it states that the force (F) needed to extend or compress a spring by some distance (x) scales linearly with respect to that distance,
(k – so called Hooke’s constant)

Equality of these 2 forces gives us the basic equation for a simple harmonic oscillator:

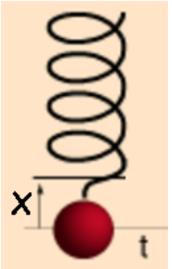
$$m\frac{d^2x(t)}{dt^2} = -kx(t)$$

$$\frac{d^2x(t)}{dt^2} = -\frac{k}{m}x(t)$$

a homogeneous linear differential equation of second order with constant coefficients

1. Simple harmonic oscillator

an oscillating spring with a mass



x – distance,
t – time,

$$\frac{d^2x(t)}{dt^2} = -\frac{k}{m}x(t)$$

homogenous LDE with constant coefficients

Solution:

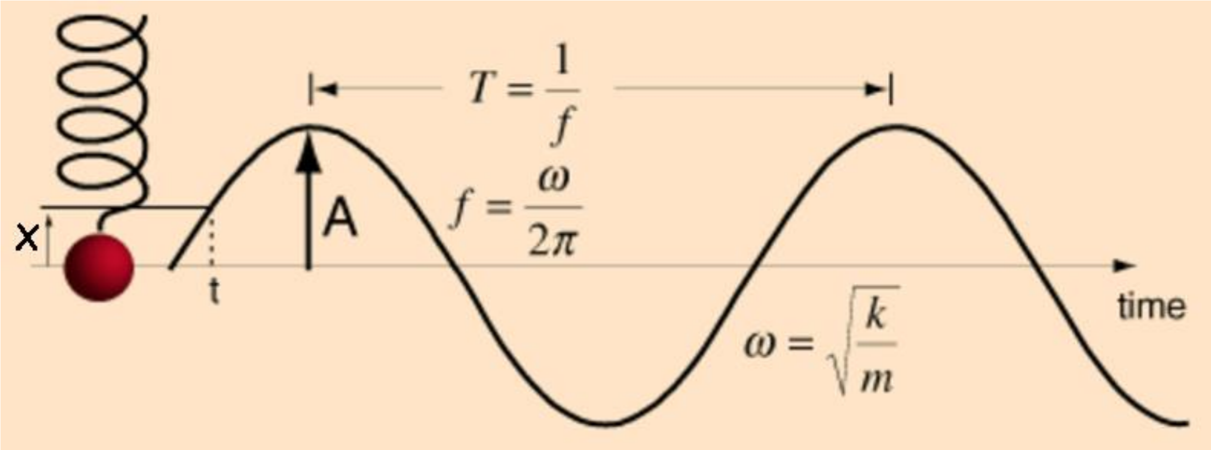
$$x(t) = C_1\cos(\omega t) + C_2\sin(\omega t) = A\cos(\omega t - \varphi)$$

Periodic motion

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} = 2\pi f$$

$$A = \sqrt{C_1^2 + C_2^2}$$

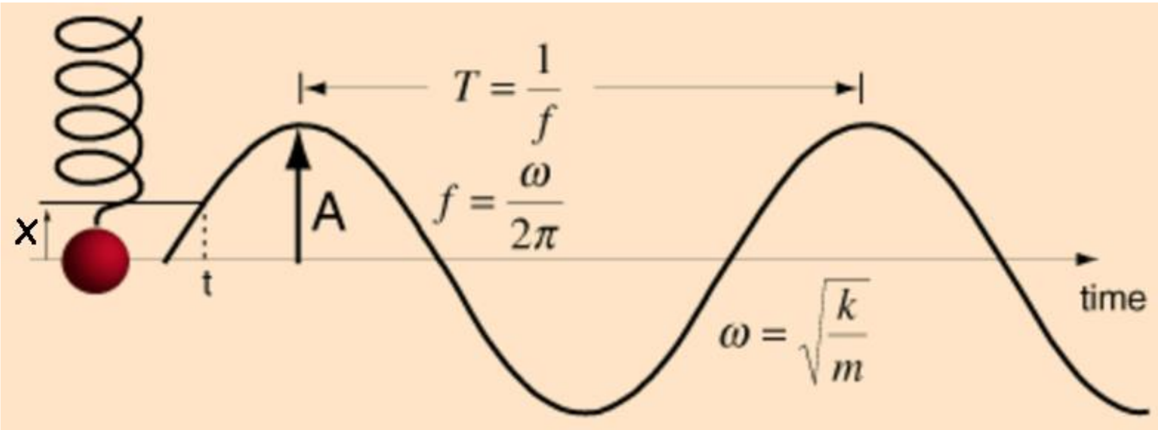
$$\tan\varphi = \frac{C_1}{C_2}$$

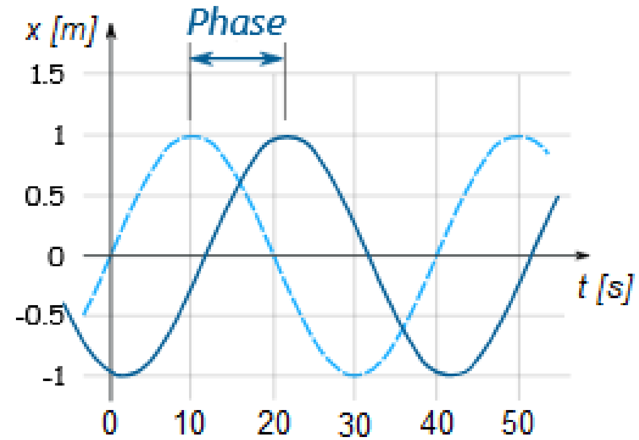


A – amplitude, φ – phase , ω – angular frequency , f – frequency , t – time

1. Simple harmonic oscillator

an oscillating spring with a mass





Speed $$\left|\vec{v}(t)\right| = \frac{dx(t)}{dt} = -A\omega\sin(\omega t - \varphi)$$

Acceleration $$\left|\vec{a}(t)\right| = \frac{d^2x(t)}{dt^2} = -A\omega^2\cos(\omega t - \varphi)$$

A – amplitude, φ – phase , ω – angular frequency , f – frequency , t – time

1. Simple harmonic oscillator

an oscillating spring with a mass

$$x(t) = C_1 \cos(\omega t) + C_2 \sin(\omega t) = A\cos(\omega t - \varphi)$$

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} = 2\pi f$$

André Kuipers uses a BMMD

Example of an astronaut at the ISS, using a special oscillator device for the estimating of his mass…
(BMMD = Body Mass Measurement Device)

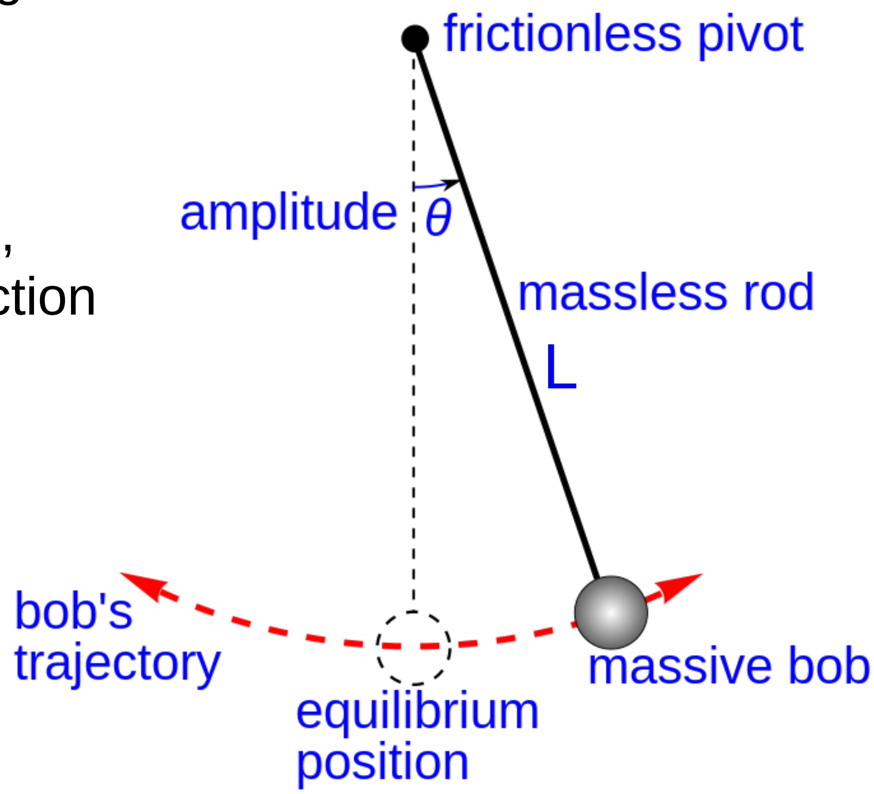
https://www.youtube.com/watch?v=6zEMEuvBuXQ

1. Simple harmonic oscillator

Next example – mathematical pendulum (or simple gravity pendulum)

Basic assumptions:

- the rod or cord on which the bob swings is massless,
- the bob is a point mass,
- motion occurs only in two dimensions (i.e. the bob does not trace an ellipse),
- the motion does not lose energy to friction or air resistance,
- the gravitational field is uniform,
- the support does not move.



period T:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

2. Damped oscillator

Friction (damping) force F_f:

$$F_f = -cv = -c\frac{dx}{dt}$$

c - coefficient of damping force

$$-kx - c\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

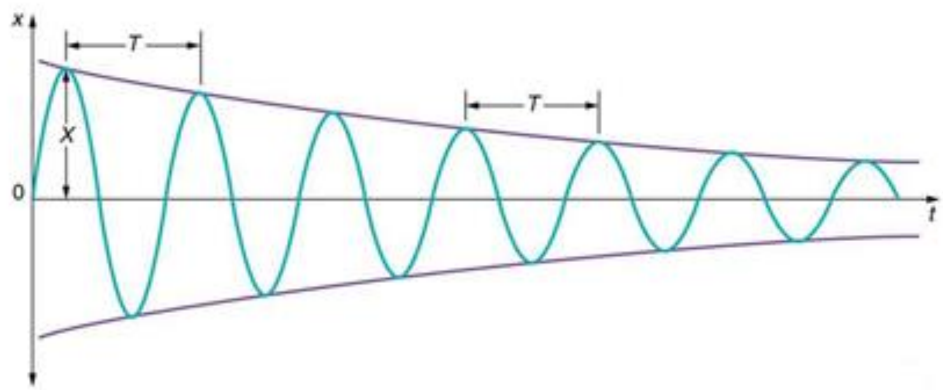
(next type of differential equation)

$$\frac{d^2x}{dt^2} + 2\zeta\omega\frac{dx}{dt} + \omega^2 x = 0$$

$$\omega = \sqrt{\frac{k}{m}}; \quad \zeta = \frac{c}{2\sqrt{mk}}$$

$\zeta > 1$ over-damped $\quad$ $\zeta = 1$ critically damped $\quad$ $\zeta < 1$ under-damped

Solution: $x(t) = Ae^{-\zeta\omega_0 t}\sin\left(\sqrt{1-\zeta^2}\,\omega t + \varphi\right)$



3. Driven oscillator

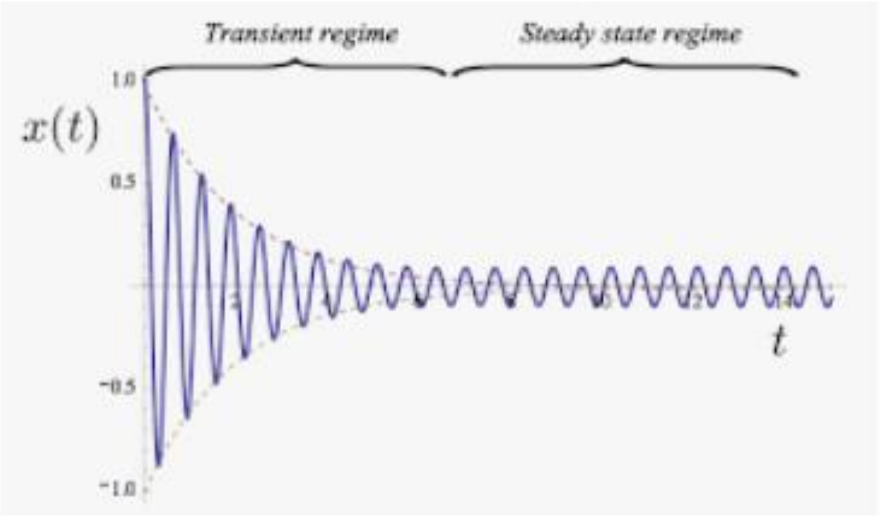
Externally applied force F_{ext}

$$F = F_{ext} - kx - c\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

(next type of differential equation…)

$$\frac{d^2x}{dt^2} + 2\zeta\omega_0\frac{dx}{dt} + \omega_0^2 x = \frac{F}{m}$$

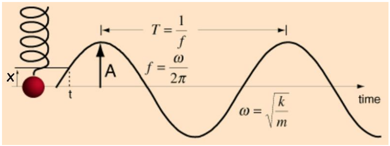
Solutions are depend on external force (e.g. can be sinusioidal).



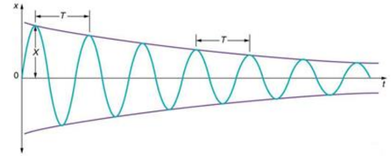
SUMMARY – HARMONIC OSCILLATORS SOLUTIONS

HARMONIC OSCILLATOR

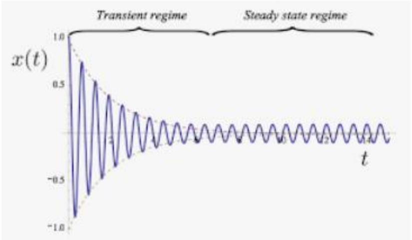
- **Simple harmonic oscillator -** F is the only acting force

  solution: 

- **Damped oscillator –** Friction (damping) occurs

  solution: 

- **Driven oscillator –** Damped oscillator further affected by an external force

  solution: 

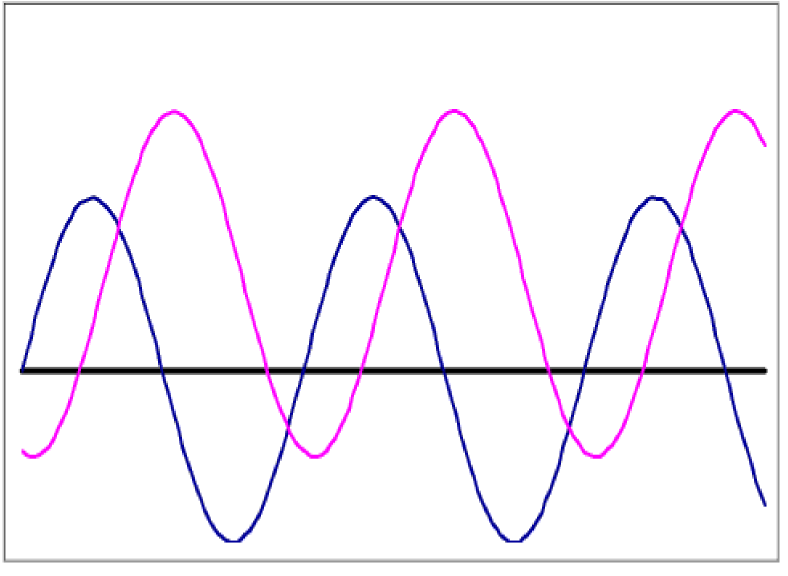
Combinations of oscillations

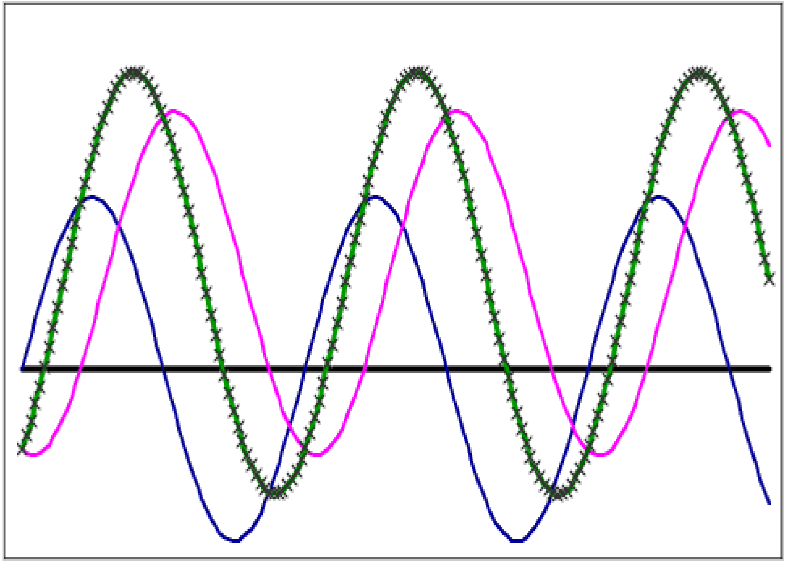
Oscillation 1 $x_1(t) = A_1 \cos(\omega_1 t - \varphi_1)$

Oscillation 2 $x_2(t) = A_2 \cos(\omega_2 t - \varphi_2)$

$$x = x_1 + x_2 = A_1 \cos(\omega_1 t - \varphi_1) + A_2 \cos(\omega_2 t - \varphi_2)$$

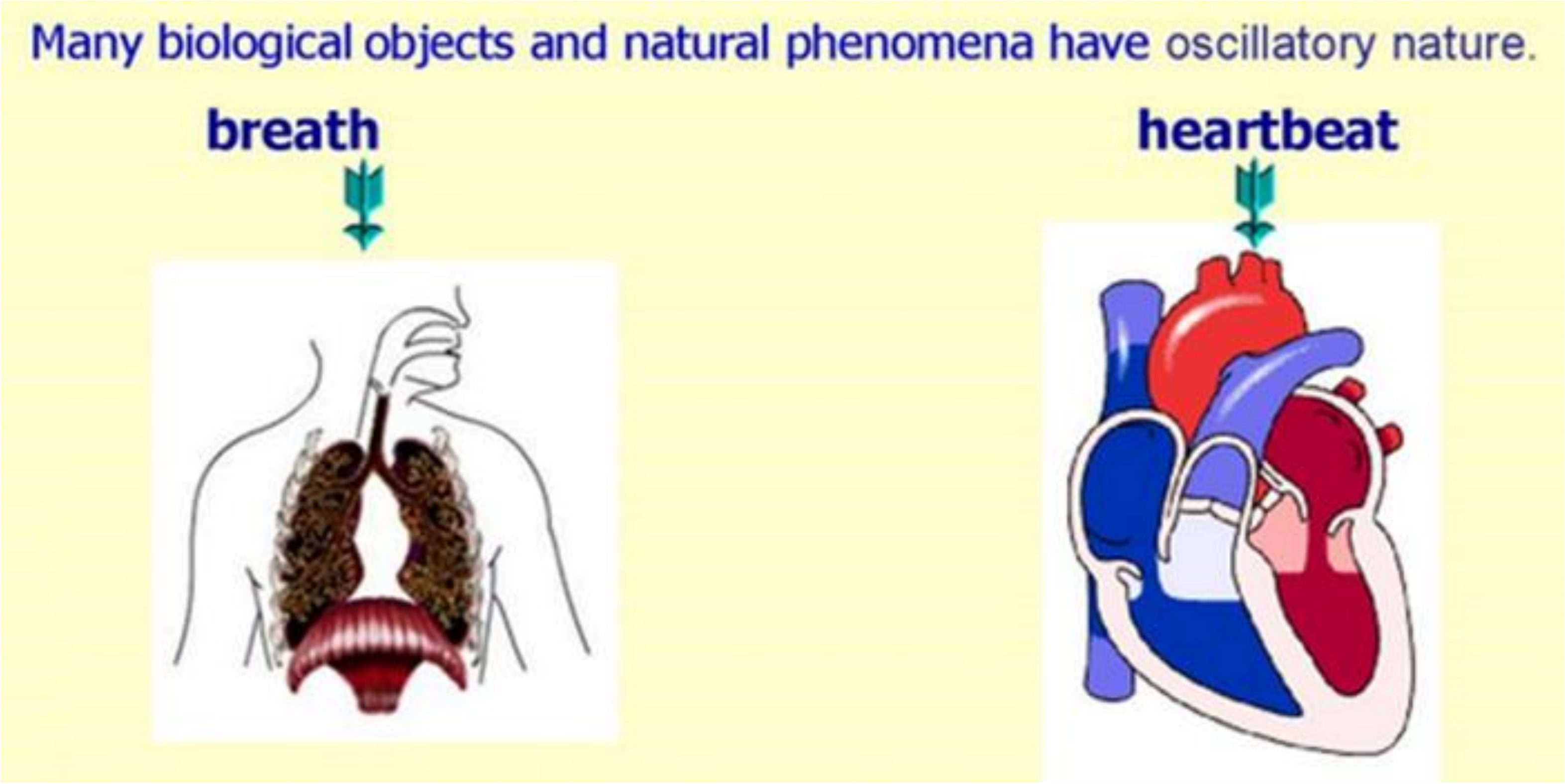
Maximum possible displacement $x_{max} = A_1 + A_2$





Comment: We will come back to this topic during the so called interference of waves.

oscillators in biology



Comment: In biology, there is very important so called circardian rhytm.

Lecture 3: Oscillators, waves

Content:

- harmonic oscillator
- mathematical pendulum
- damped oscillator
- driven oscillator
- waves
- Huygens principle, Doppler effect
- flow of liquids and air

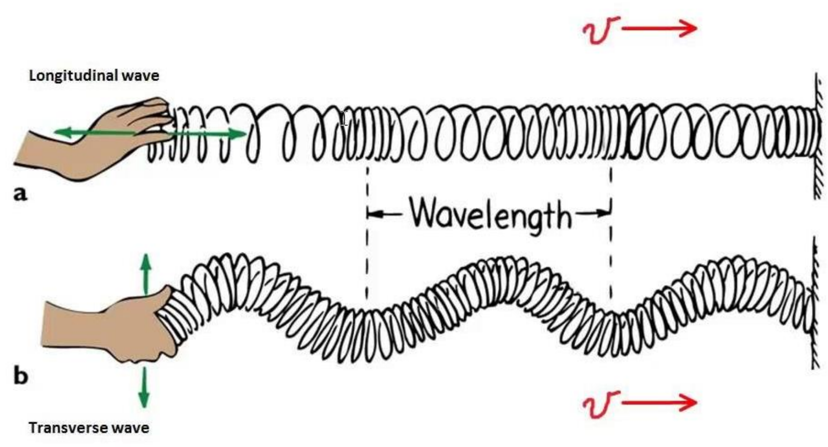
Waves

In physics, a wave is a propagating dynamic disturbance (change from equilibrium) of one or more quantities, sometimes as described by a wave equation.

– Oscillation accompanied by the transfer of energy which travels through mass,
– No (or very little) mass transport.

waves
- mechanical – through medium which is deformed, e.g. sound waves, body waves
- electromagnetic – no medium, periodic oscillations of electric and magnetic fields, e.g. visible light, radio waves, mobile phones...

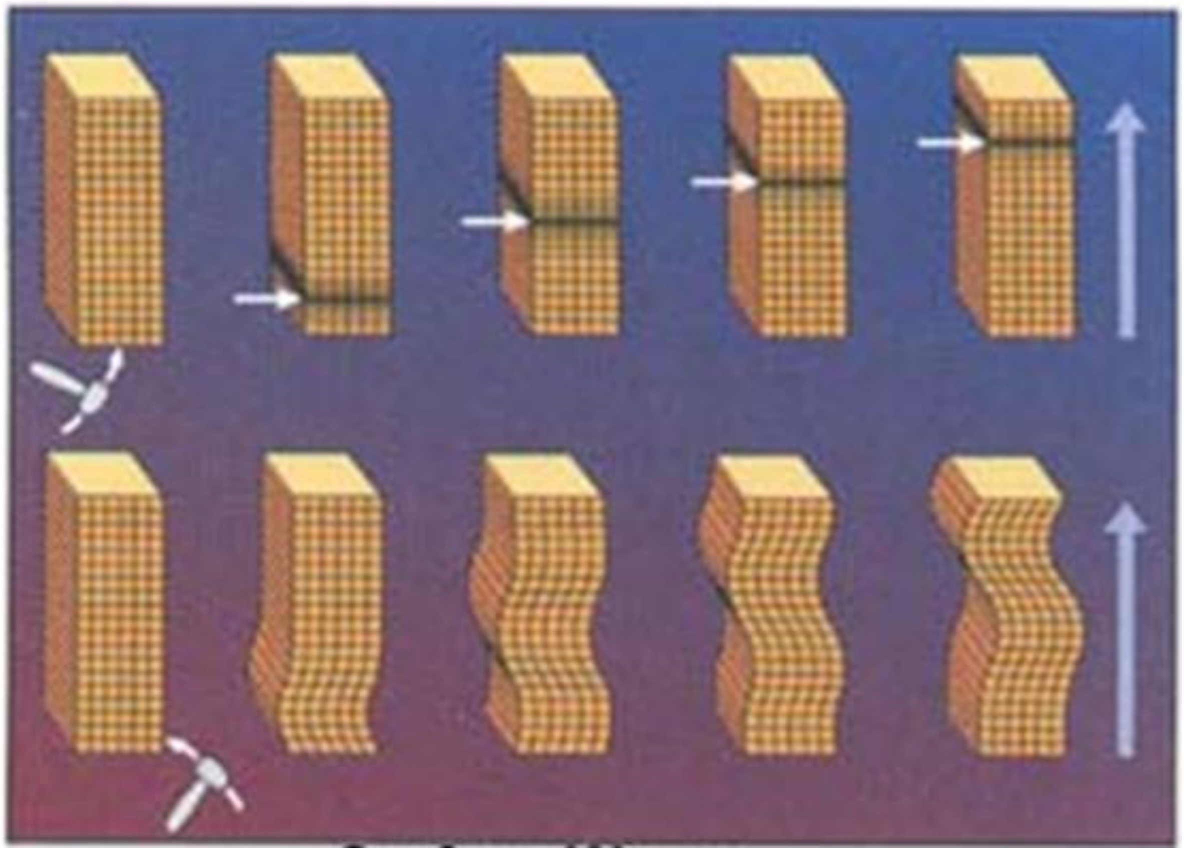
waves
- transverse – oscillations are perpendicular to the energy transfer
- longitudinal – oscillations are parallel to the energy transfer



Waves

waves

- transverse – oscillations are perpendicular to the energy transfer
- longitudinal – oscillations are parallel to the energy transfer



longitudinal

transverse

Comment: Transverse waves can not propagate in liquids or gases.

Wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

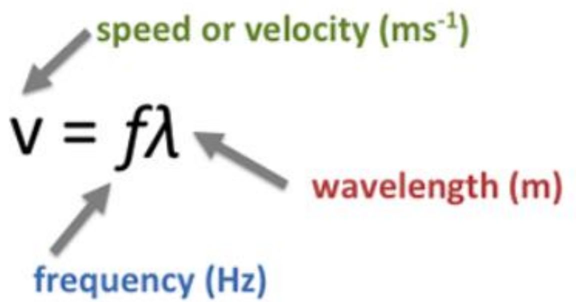
$u = u(x_1, x_2, \ldots, x_n, t)$ - scalar function whose values can model, for example, the mechanical displacement of a wave

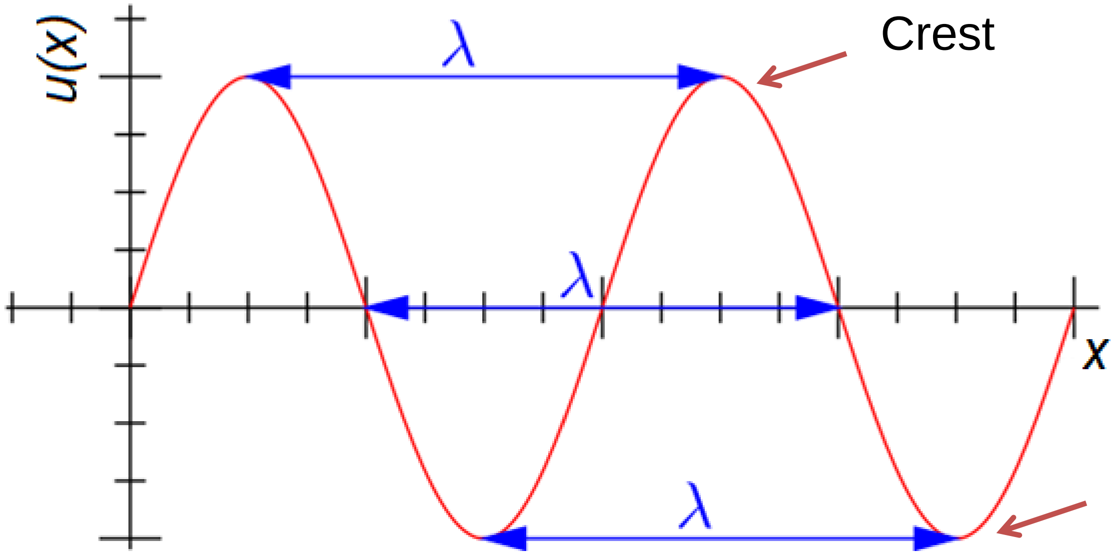
One space dimension case: $$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

Now the searched solution (u) is not only a function of time (t), but also function of space (x).

Most simple solution: $$u = A \sin\left(\frac{2\pi}{\lambda} x\right)$$

One important consequence:

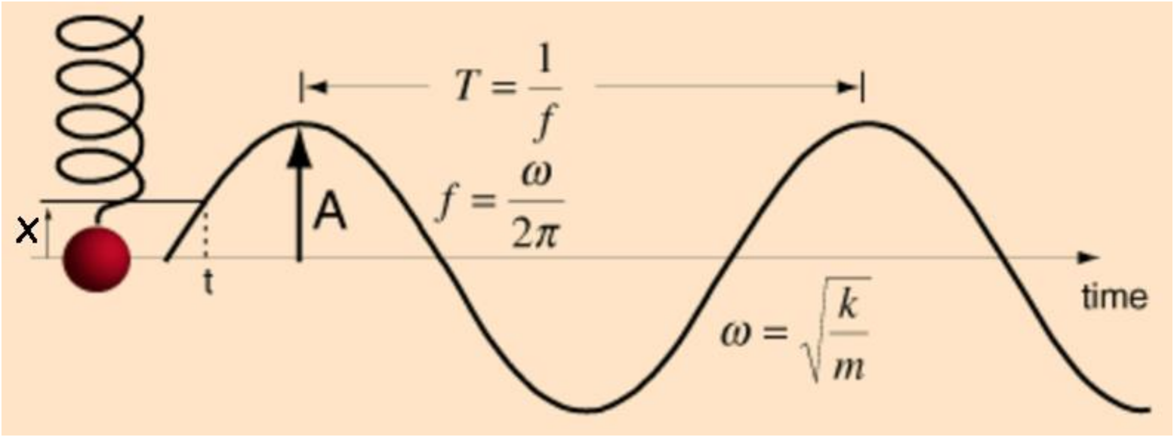




λ – wavelength (meters)

$$\lambda = vT = \frac{v}{f}$$

Comment: back to the harmonic oscillator



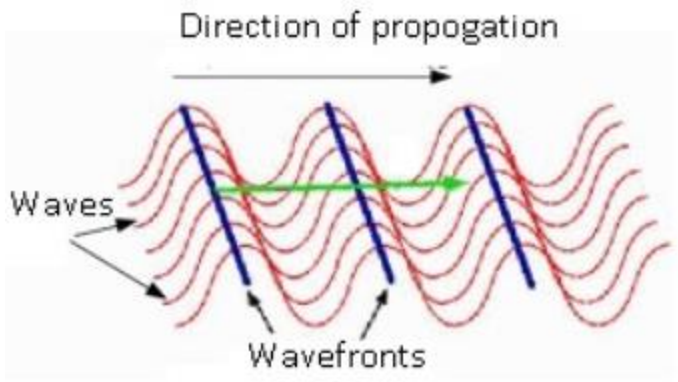
T – period (seconds)

$$T = \frac{1}{f}$$

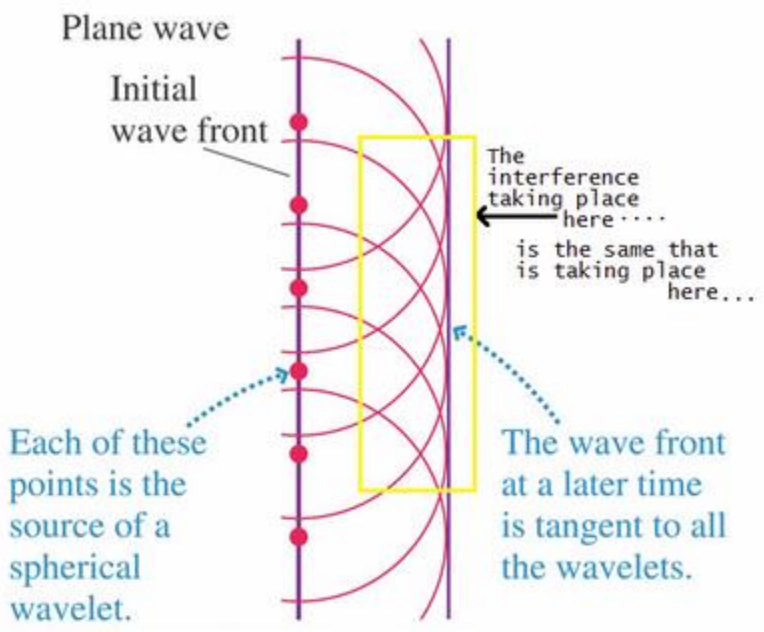
A – amplitude

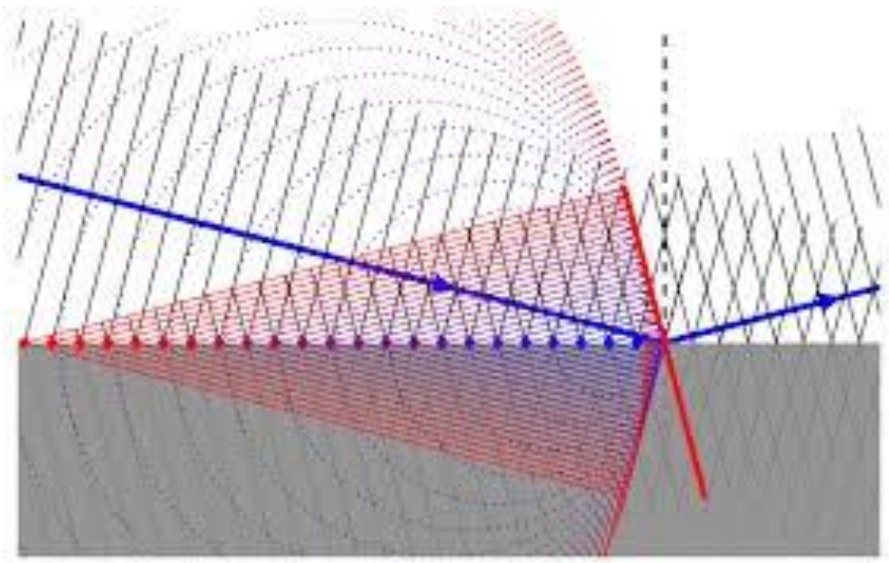
Wavefronts, Huygens principle

Waves can propagate in packages, called as wavefronts.



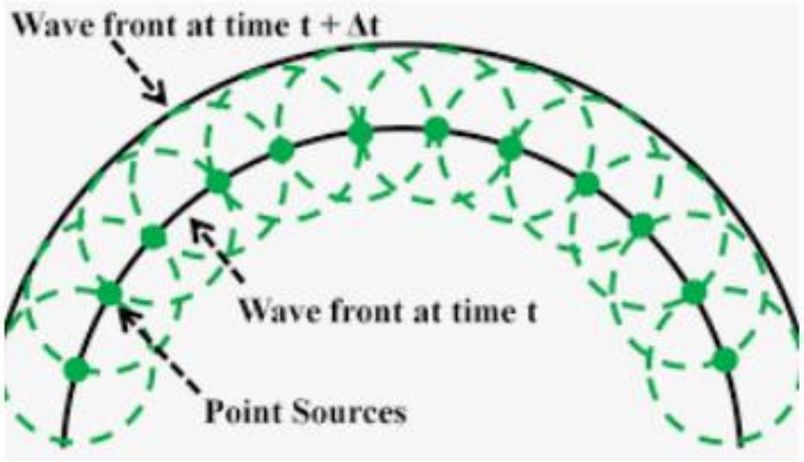
<u>Huygens principle:</u> Every point on a wave-front may be considered a source of secondary spherical wavelets which spread out in the forward direction. The new wave-front is the tangential surface to all of these secondary wavelets.

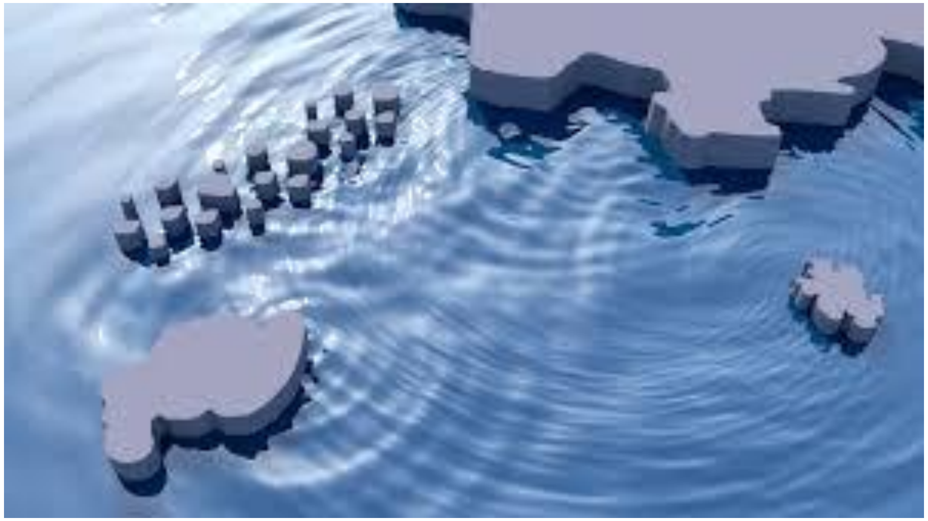




Wavefronts, Huygens principle

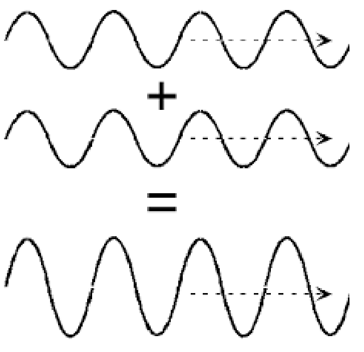
<u>Huygens principle:</u> Every point on a wave-front may be considered a source of secondary spherical wavelets which spread out in the forward direction. The new wave-front is the tangential surface to all of these secondary wavelets.

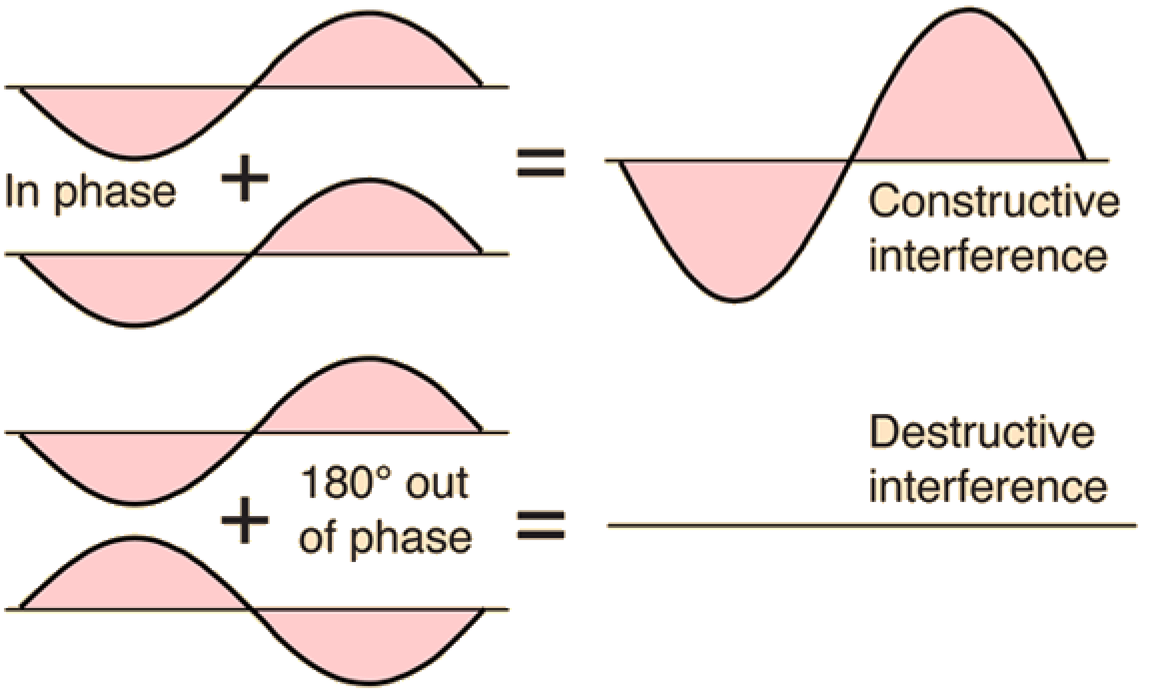




Interference

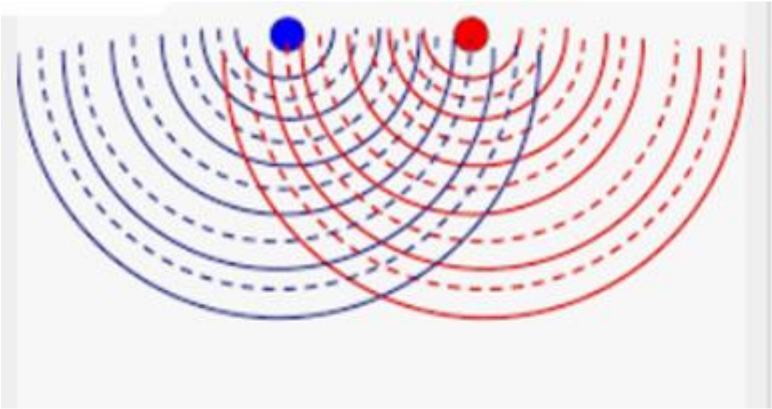
The principle of superposition of waves states that when two or more propagating waves of same type are incident on the same point, the total displacement at that point is equal to the point wise sum of the displacements of the individual waves.

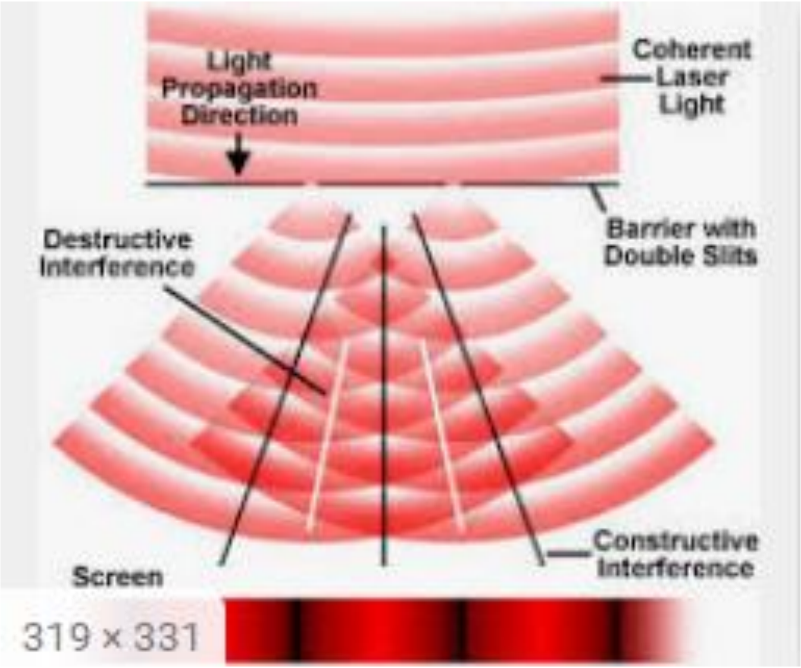


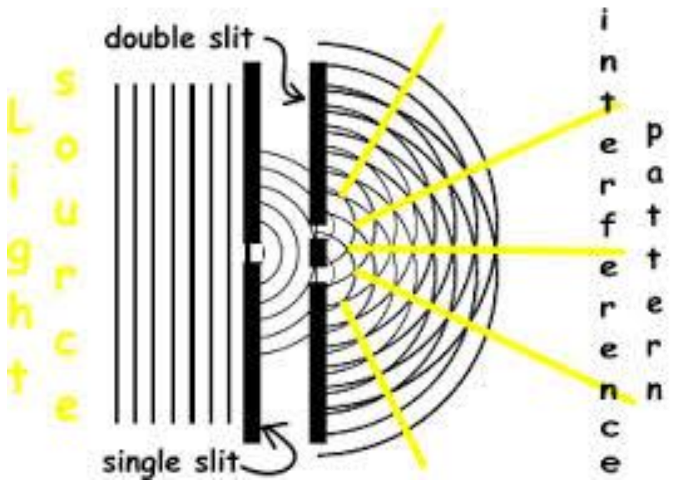


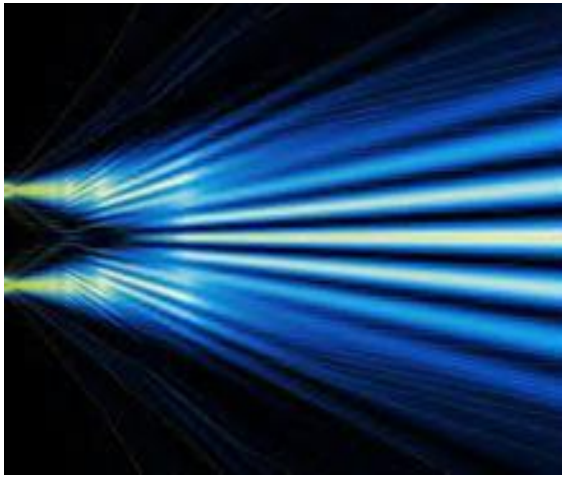
Interference

The principle of superposition of waves states that when two or more propagating waves of same type are incident on the same point, the total displacement at that point is equal to the point wise sum of the displacements of the individual waves.



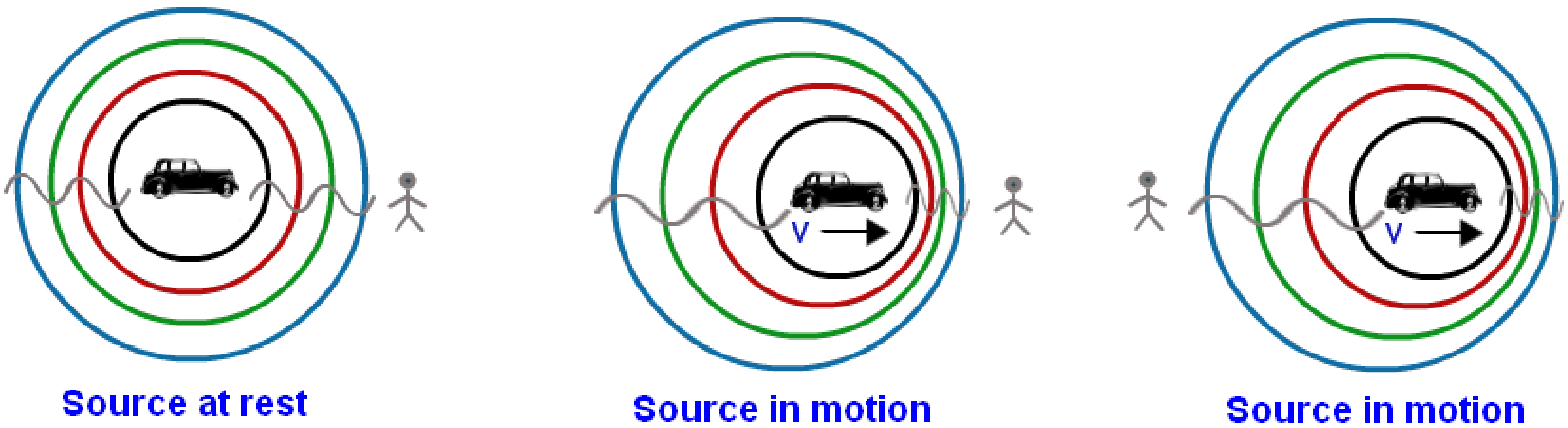






Doppler effect

It is the change in frequency of a wave (or other periodic event) for an observer moving relative to its source. The received frequency is higher during the approach, identical at the instant of passing by, and lower during the recession.



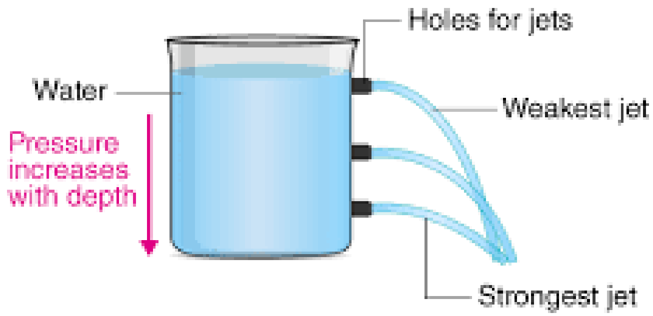
Lecture 3: Oscillators, waves

Content:

- harmonic oscillator
- mathematical pendulum
- damped oscillator
- driven oscillator
- waves
- Huygens principle, Doppler effect
- flow of liquids and air

flow of liquids and air

To understand the basic rules, which are valid for the flow of ideal liquids and air, we have to start with the hydrostatic/airstatic pressure.



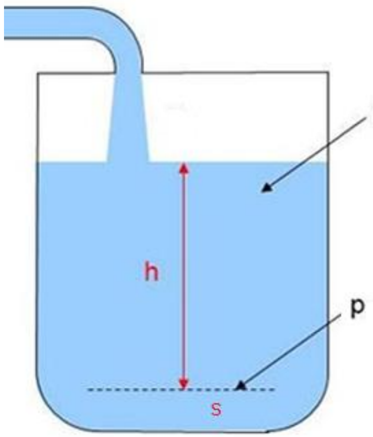
pressure in general: force/area

$$p = \frac{|\vec{F}|}{s} = \frac{F}{s}$$

Hydrostatic/airstatic pressure: comes from the weight of the liquid(water)/air column above some area.

Weight is gravity force of the mass of liquid(water)/air:

$F_G = mg = \rho Vg = \rho shg.$



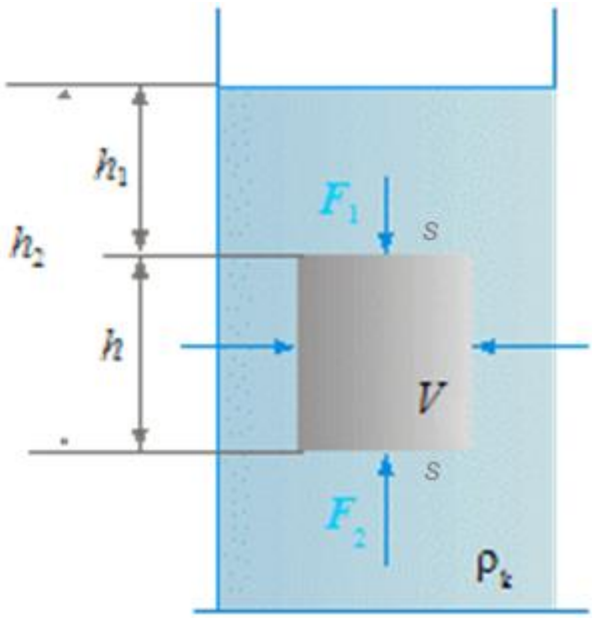
Hydrostatic/airstatic pressure:

$$p = F_G/s = \rho hg.$$

where ρ is the density of the liquid(water) or air.

Beside hydrostatic pressure we know also hydrodynamic pressure.

Lifting force (F_{lift}), is the difference of hydrostatic pressures.



- On the upper base of the prism force F_1 is acting:
  $$F_1 = h_1 \rho g s$$
- On the bottom base force F_2 is acting:
  $$F_2 = h_2 \rho g s$$
  $$F_2 > F_1$$
- Result of these two acting forces is the so called **hydrostatic lifting force F_{lift}:**
-
  $$F_{lift} = F_2 – F_1$$
  $$F_{lift} = (h_2 – h_1) \rho g s$$
  $$F_{lift} = h \rho g s$$
  $$F_{lift} = \rho g V$$ ← **basics of the law of Archimedes**

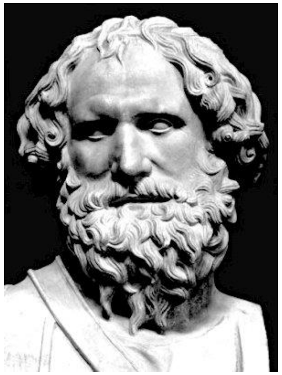
Important: ρ is the density of the liquid (water) and the body!

law of Archimedes

- Any body completely or partially submerged in a fluid (gas or liquid) at rest is acted upon by a lifting (upward) force, the magnitude of which is equal to the weight of the fluid displaced by the body.

- magnitude of the lifting (buoyant) force:

$$F_{lift} = \rho\, g\, V$$



Archimedes
(approx. 287 – 212 BC)

flow of liquids and air

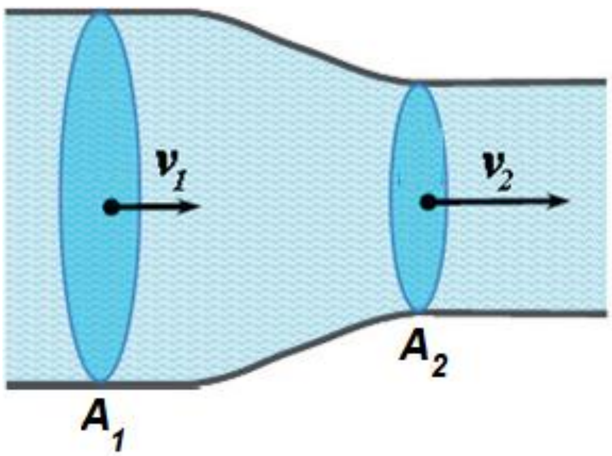
Continuity equation (equation of the volume flow continuity)

For an ideal fluid (can not change the volume) is valid a constant volume flow:

$$|v|_1 A_1 = |v|_2 A_2 \quad \Rightarrow \quad |v| A = \text{const.}$$

It „speaks“ about the so called mass conservation.

here: A – area, v – speed (velocity size)



Volume flow Q is defined as the volume, which pass thru some cross-section in some time.
unit: [$m^3 s^{-1}$]

$$Q = \frac{V}{t} = \frac{d \cdot A}{t} = |v| A$$

here: Q – volume flow, V – volume, t – time,
A – area, v – speed (velocity size),
d – distance

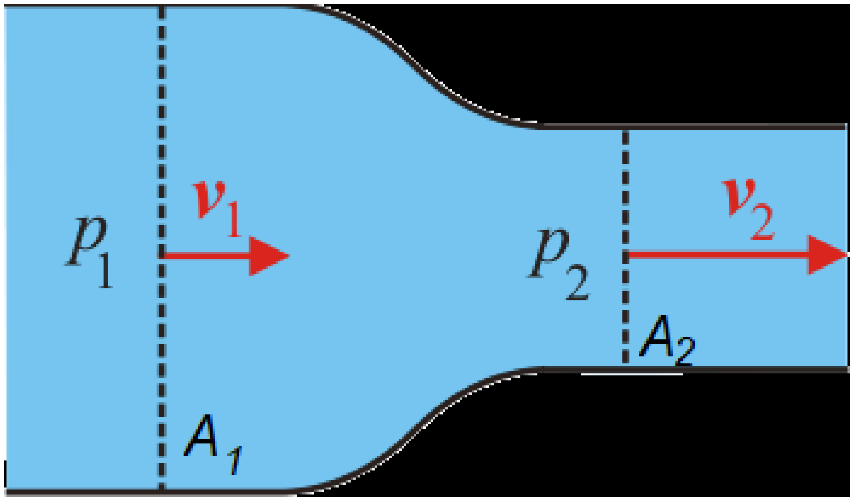
flow of liquids and air

Bernoulli equation

For an ideal fluid (can not change the volume) following equation is valid:

$$p_1 + \rho h_1 g + \frac{1}{2}\rho v_1^{\,2} = p_2 + \rho h_2 g + \frac{1}{2}\rho v_2^{\,2} = \text{const.}$$

hydrostatic pressure　　　hydrodynamic pressure

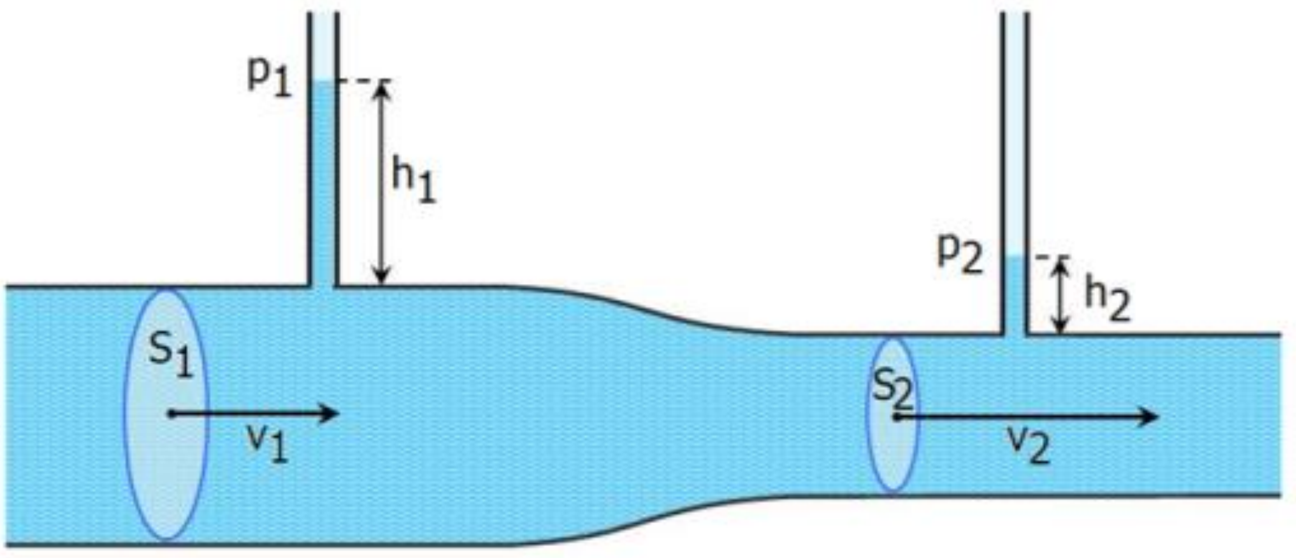


flow of liquids and air

Bernoulli equation – so called hydrodynamic paradox

$$p + \rho hg + \frac{1}{2}\rho v^2 = const.$$

It is a phenomenon, when in a narrower pipe (with higher flow speed) occurs smaller pressure.

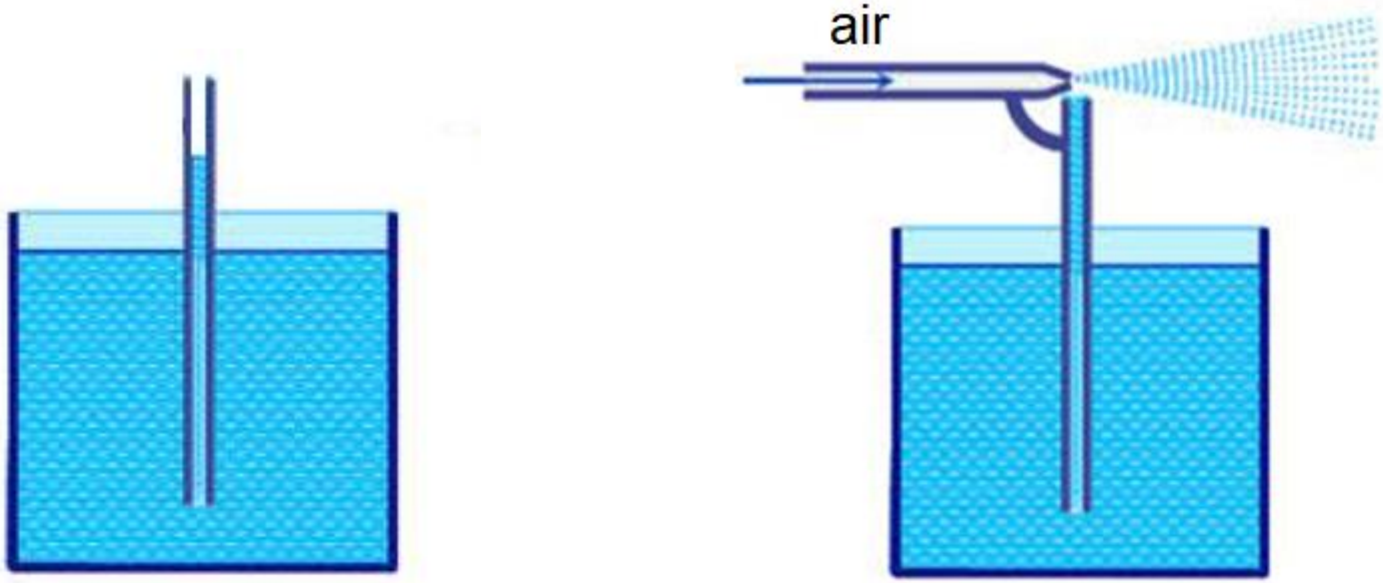


This is valid also for air (gas) – so called aerodynamic paradox.

flow of liquids and air

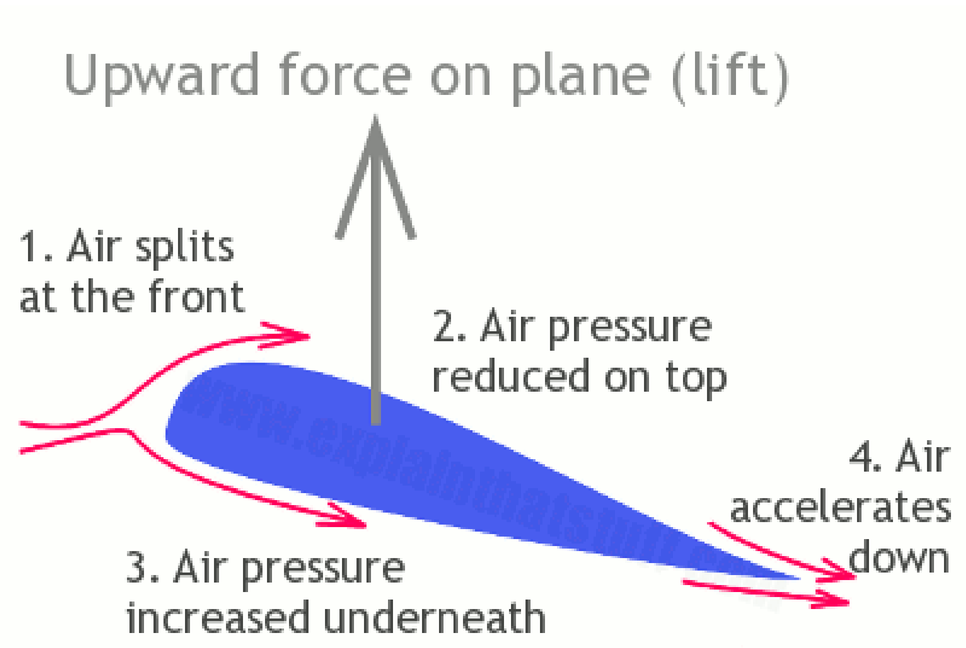
Bernoulli equation – so called hydrodynamic paradox

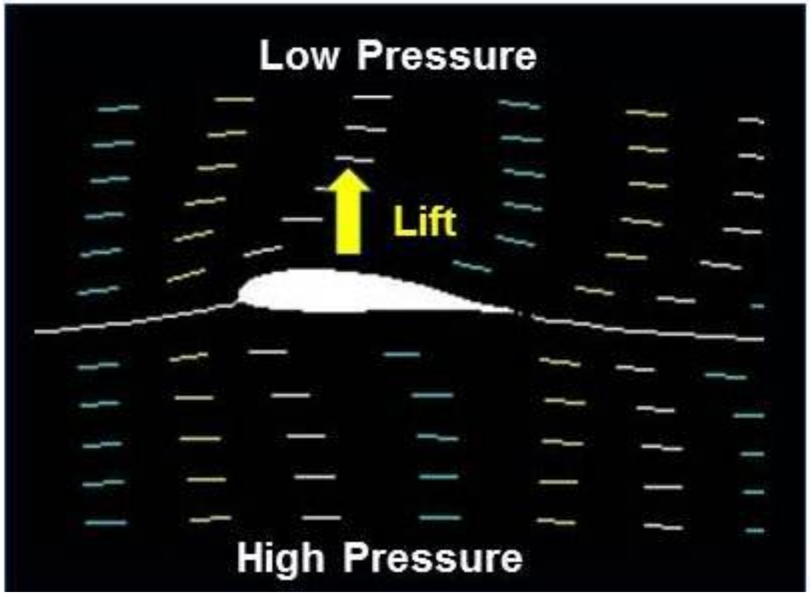
$$p + \rho hg + \frac{1}{2}\rho v^2 = const.$$



Principle of spray effect.

Why can a heavy aircraft stay in air and fly?





flow of liquids and air

drag - hydrodynamic force acting opposite the flow

Drag is a force acting opposite to the relative motion of any object moving with respect to a surrounding fluid. So called Newton formula is valid:

$$F = \frac{1}{2} C A \rho v^2$$

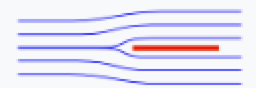
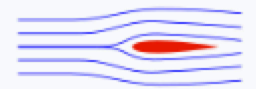
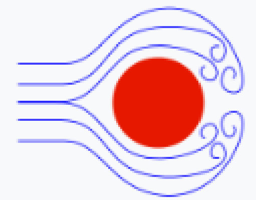
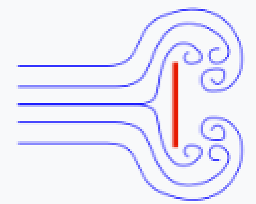
where:

C – so called drag coefficient (dimensionless), depending from the form of the object,

A – area of cross-section,

ρ – density of the liquid,

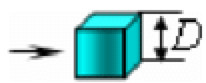
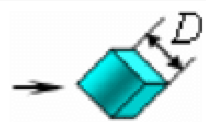
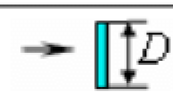
v – velocity (speed).

Shape and flow	Form Drag
	≈0%
	≈10%
	≈90%
	≈100%

drag - hydrodynamic force acting opposite the flow

Drag is a force acting opposite to the relative motion of any object moving with respect to a surrounding fluid. So called Newton formula is valid:

$$F = \frac{1}{2} C A \rho v^2$$

Type of body		Reference area S	Drag coefficient C_D
Cube		$S = D^2$	1.05
		$S = D^2$	0.8
Solid hemisphere		$S = \pi D^2 / 4$	$\rightarrow$ 0.42 $\leftarrow$ 1.17
Hollow hemisphere		$S = \pi D^2 / 4$	$\rightarrow$ 0.38 $\leftarrow$ 1.42
Thin disk		$S = \pi D^2 / 4$	1.1

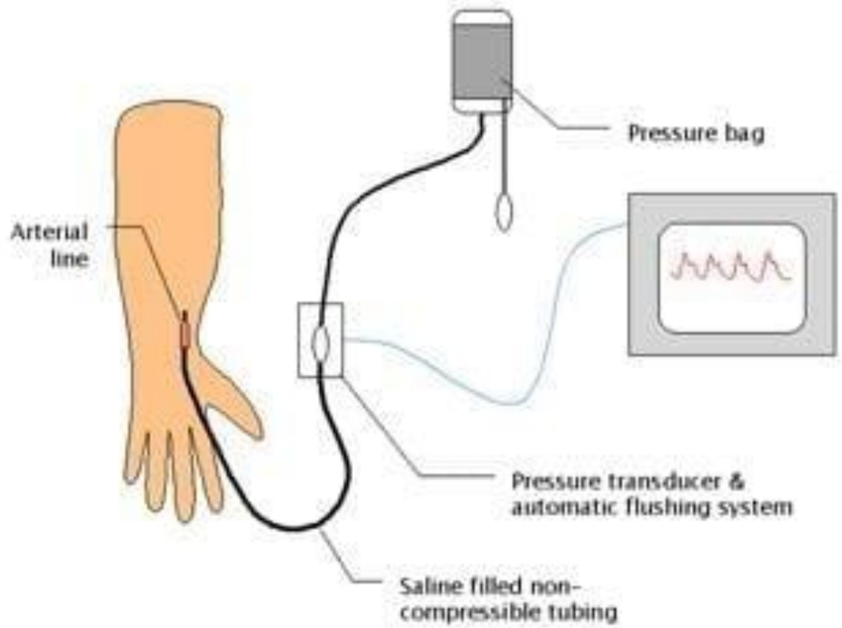
(C is without physical unit)

Blood pressure

Blood pressure is measured as two numbers: **Systolic** blood pressure (the first and higher number, ideally **120**) measures pressure inside your arteries when the heart beats. **Diastolic** blood pressure (the second and lower number, ideally **80**) measures the pressure inside the artery when the heart rests between beats.

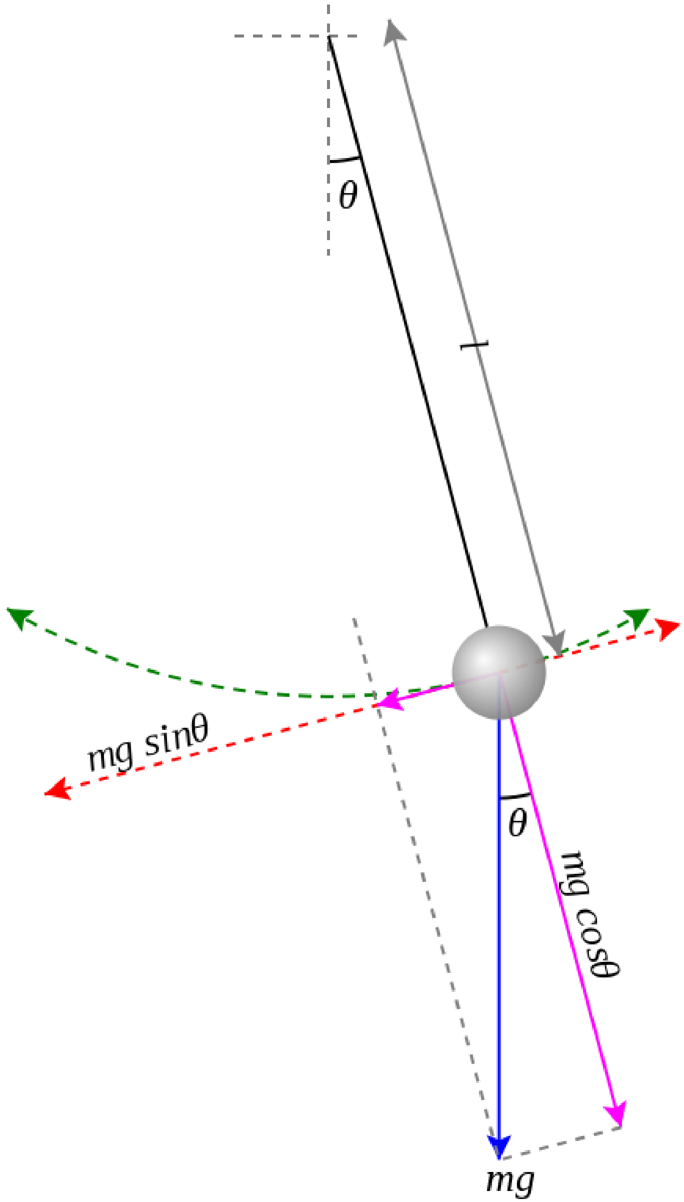
Physical unit – an older unit is still used: [mm Hg].

Conversion: 1 mmHg = 133.322 Pa.



solution for the mathematical pendulum (1/2)

(by means of the solution of a differential equation of second order)



Equality of 2 equations for accelerations gives us the basic equation for the pendulum:

$$F = m \cdot a$$ **a)**

$$F = -mg\sin\theta = m \cdot a \Rightarrow a = -g\sin\theta$$

b)

$$s = l \cdot \theta \rightarrow v = \frac{ds}{dt} = l\frac{d\theta}{dt} \rightarrow a = l\frac{d^2\theta}{dt^2}$$

$$l\frac{d^2\theta}{dt^2} = -g\sin\theta \Rightarrow \frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$$

l – length of the pendulum

θ – angular displacement (amplitude)

solution for the mathematical pendulum (2/2)

(by means of the solution of a differential equation of second order)

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$$

assumption: $\theta \ll 1 \rightarrow \sin\theta \approx \theta$

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0$$

boundary conditions $\theta(0) = \theta_0; \quad \left.\frac{d\theta}{dt}\right|_{t=0} = 0$

Solution: $$\theta(t) = \theta_0 \cos\left(\sqrt{\frac{g}{l}}t\right)$$ **$\theta_0 \ll 1$**

Period: $$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\omega = 2\pi f \ , \ f = \frac{1}{T}$$